Turbulence Model for Nonequilibrium Adverse Pressure Gradient Flows

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Theme

A METHOD is presented for calculating attached turbulent boundary-layer flows with moderate-to-strong adverse pressure gradients by employing a modified algebraic eddy viscosity turbulence model that has been corrected for variable pressure gradient. The modifications are based on correlations obtained from the extensive amounts of data available for incompressible equilibrium boundary-layer flows. A detailed analysis of available compressible boundary-layer flows validated the trends predicted by these correlations. To extend the modified eddy-viscosity model to predict compressible nonequilibrium boundary-layer flows, a mechanism is required to account for the effects of turbulence memory. By means of a boundary layer finite-difference code employed with the new turbulence model, several experimental nonequilibrium compressible boundary-layer flows are computed. Various techniques are used to account for turbulence memory. Although previous techniques fail, a new turbulence memory expression is developed which gives good agreement between the predictions and experimental results. Finally, a correlation is developed for the lag lengths employed in the turbulence memory expression.

Contents

To adequately test a proposed turbulence model, comparisons should be made with experimental flows over a wide range of Mach and Reynolds numbers. The experimental flows should be thoroughly documented in the sense that surface and mean flowfield quantities should be measured. The upstream flow conditions must be measured, and a fully developed equilibrium turbulent boundary-layer verified ahead of the variable pressure gradient portion of the flowfield. Fluctuating flowfield measurements are desirable. Unfortunately, experiments meeting these criteria are extremely rare for compressible adverse pressure gradient flows. The experimental flows chosen for the present comparisons are those of Zwartz,[1] Peake et al.,[2] the low Reynolds number case of Sturek and Danberg,[3] Lewis et al.,[4] and the nonseparated case of Kussoy and Horstman.[5] These flows include both moderate and extremely strong adverse pressure gradients at Mach numbers from 3.5 to 6.7 and momentum thickness Reynolds numbers from 5 to 50×10^3. Well-documented experimental flowfields are not available for Mach numbers from 0.5 to 3.

The experimental boundary-layer flows were computed using a finite-difference boundary-layer program. The baseline turbulence model employed was a two-layer eddy-viscosity model. In the inner layer, a mixing-length formulation for the eddy viscosity is used

$$\epsilon_{\text{inner}} = \ell^2 \left| \frac{du}{dy} \right|$$

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where

$$\ell = Ky[1-\exp(-y^+/A^+)]$$

with $K=0.4$ and $A^+=26$. In the outer region, the eddy viscosity is represented by the Escudier model.

$$\epsilon_{\text{outer}} = \ell^2{}_{\max} \left| \frac{du}{dy} \right|$$

where $\ell_{\max} = C\delta$. For each experimental flow, a value of C was used to best match the upstream velocity profile.

The calculations employing the baseline turbulence model are compared with two of the test flows on Figs. 1 and 2. These comparisons show that the baseline model is inadequate for predicting the experimental values of skin friction, and a new model must be devised to include the effects of pressure gradient.

Corrections for the effects of variable pressure gradients based on correlations of incompressible equilibrium flows have been developed by Kays[6] for A^+ and Glowacki and Chi[7] for K. These corrections are

$$A^+ = 26/(1+30.18p^+)$$

$$K = 0.4 + 0.182257[1-\exp(-0.32068\beta)]$$

A detailed analysis of the five experimental test flows[1-5] was performed using a new technique for obtaining the shear stress profiles by integrating the mean momentum-and-energy equations. These results indicated the same trends as the previous incompressible relations.

The calculations employing the baseline model with the pressure gradient corrections are compared with the two test flows on Figs. 1 and 2. These comparisons show that these modifications give poorer results than the baseline model. Evidently, the full effect of the pressure gradient on the shear stress is not felt as soon as the pressure gradient is applied, because turbulence has a large memory and is influenced by the past history of the flow. By examining the comparisons, it can be seen that, if the increases in skin friction due to the

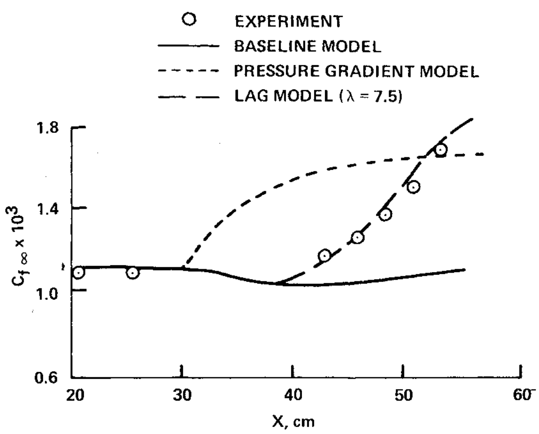

Fig. 1 Comparison of measured and computed skin-friction distributions for the Sturek and Danberg flow.

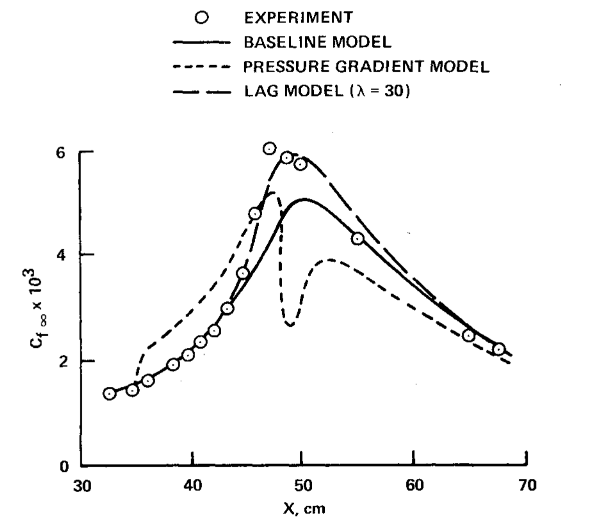

Fig. 2 Comparison of measured and computed skin-friction distributions for the Lewis, Gran, and Kubota flow.

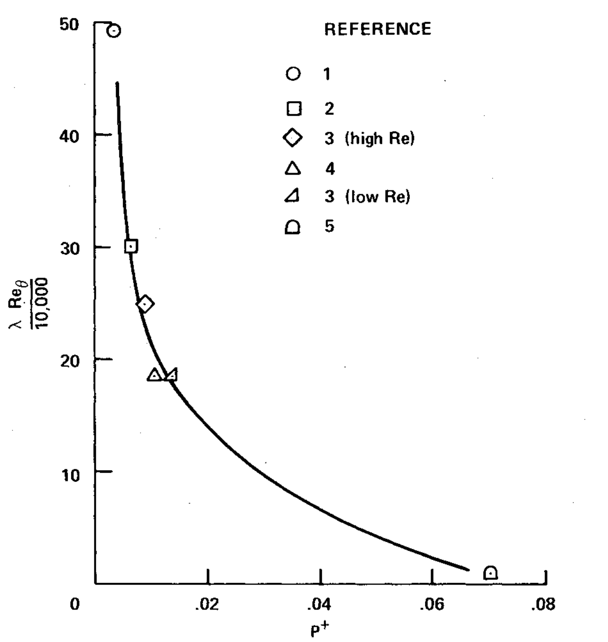

Fig. 3 Variation of the normalized lag-length parameter with pressure gradient.

pressure gradient correction were "delayed" or shifted downstream, the agreement with the data would improve.

To account for turbulence memory effects with simple eddy viscosity descriptions, two methods [8,9] have been proposed. Both these methods were applied to the present test flows with little success.

It is now proposed that a new lag model be used to express K and A^+ as functions of weighted averages of β and p^+ upstream of the region of interest. A Gaussian weighting function, centered about a point several boundary-layer thicknesses ($\lambda\delta$) upstream of each axial grid location, was chosen to obtain the weighted averages. The expression for A^+ now becomes

$$A^+(x) = 26/[1 + 30.18\overline{p^+(x)}]$$

where

$$\overline{p^+(x)} = \int_{x-2\lambda\delta}^{x} w(\xi) p^+(\xi) d\xi$$

$$W(\xi) = \frac{1}{\sqrt{2\pi}\sigma} \exp\{-[\xi - (x - \lambda\delta)]^2 / 2\sigma^2\}$$

$$\sigma = \lambda\delta / 3$$

where λ is a lag length parameter. A similar expression was used for $\overline{\beta(x)}$ for calculating $K(x)$.

Solutions were obtained for each test flow for several values of the lag parameter λ. For each flow, there is a value of λ which provides the best agreement with the data. Results for two test flows are shown in Figs. 1 and 2. There is excellent agreement between the calculations and experiment. Similar results were obtained for the other test flows.

To make the proposed turbulence model a "predictive" model, a method must be established to determine the lag parameter λ a priori. On Fig 3, the "best fit" values of λ, normalized by the average momentum thickness Reynolds number for each flow, are plotted versus the maximum value of p^+ obtained for each flow. The correlation with the maximum values of p^+ for each flow is apparent. This represents a first attempt to provide a method for determining the lag parameter λ. Before this correlation or any other proposed predictive method can be adopted, it is clear that additional experimental data are needed for values of p^+ between 0.01 and 0.06 for a wide range of Reynolds numbers.

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